IX. *Short and eaſy Methods for finding (1.) The Quantity of Time contained in any given Number of Mean Lunations; (2.) The Number of Mean Lunations contained in any given Quantity of Time; (3.) The Number of Troy Pounds contained in any given Number of Avoirdupoiſe Pounds, and* vice verſâ; *(4.) The Quantity and Weight of Water contained in a full Pipe of any given Height, and Diameter of Bore; and conſequently, to find what Degree of Power would be required to work a common Pump, or any other Hydraulic Engine, when the Diameter of the Pump-bore, and the Height to which the Water is to be raiſed therein, are given. Communicated by Mr.* James Ferguſon, *F. R. S.*

Read Feb. 7, 1765.

SOME time ago, juſt as I was finiſhing a table for ſhewing the quantity of time contained in any given number of mean lunations within the compaſs of 6000 Julian years, intending thereby to examine how near our preſent aſtronomical tables would ſhew the times of ſome antient eclipſes which we have on record, when the proper equations of the ſolar and lunar motions were

taken

taken into the account; I was viſited by my friend William Rivet, Eſq; of the Inner Temple, who ſaid he wiſhed he had come ſooner, becauſe he could have put me upon a much ſhorter method of computation. I deſired him to ſhew me his method, which he did moſt readily and chearfully. " It was
" only to reduce the odd hours, minutes, ſeconds,
" and thirds, &c. above the integral days of a luna-
" tion, into the decimal parts of a day; which num-
" be of days and decimal parts, being nine times
" added together, will be equal to the time contained
" in nine mean lunations. And from theſe, the
" time contained in any other aſſigned number may
" be found, as follows.

A Table ſhewing the quantity of time contained in any given number of mean lunations. The mean lunation being 29 days $12^h\ 44'\ 3''\ 2'''\ 58''''$; or 29.53059085108 days.

Lun.	Days. Decimals of a Day.
1	29.53059085108
2	59.06118170216
3	88.59177255324
4	118.12236340432
5	147.65295425540
6	177.18354510648
7	206.71413595756
8	236.24472680864
9	265.77531765972

EXPLANATION.

For tens of lunations, remove the decimal point one place forward; for hundreds of lunations, two places; for thouſands, three places; and ſo on, as in the annexed example; and then the remaining decimals may be reduced into hours, minutes, ſeconds, &c. by the common method of reducing decimals to the known parts of an integer.

EXAMPLE.

EXAMPLE.

In 74212 mean lunations. Qu. How many days, hours, minutes, and ſeconds?

Lun.	Days Decim. of a Day.
70000	2067141·3595756
4000	118122.36340432
200	5906.118170216
10	295·3059085108
2	59·06118170216
74212	2191524.20824034896

Anſ. 2191524 days and .20824034896 decimal parts of a day.

And by reduction, 2191524 days contain 6000 Julian years, 24 days, and .20824034896 decimal parts of a day, contain 4 hours, 59 minutes, 51 ſeconds, 57 $\frac{969}{1000}$ thirds. But in practice, it is ſufficient to take in four or five of the decimal figures.

Having got this hint from Mr. Rivet, I reverſed it into a way of finding the number of mean lunations contained in any given quantity of time; for which purpoſe I calculated the following table, upon the above length of a mean lunation; which comes the neareſt to the truth of any length I have yet found, when carried back from the preſent times to the recorded times of antient eclipſes, if the proper equations depending upon the anomalies of the ſun and moon are applyed to the mean times of new and full moons.

A Table

A Table ſhewing the number of mean lunations contained in any given quantity of time.

Y.	Lun. Decimals of a Lunation.	H.	L. Decimals of a Lunation.	S.	L. Decimals of a Lunation.
1	12.36853003863	1	0.00141096624	1	1.00000039193
2	24.73706007726	2	0.00282193248	2	2.00000078387
3	37.10559011589	3	0.00423289872	3	3.00000117580
4	49.47412015451	4	0.00564386496	4	4.00000156774
5	61.84265019314	5	0.00705483120	5	5.00000195967
6	74.21118023176	9	0.00846579744	6	6.00000235161
7	86.57971027039	7	0.00987676368	7	7.00000274354
8	98.94824030902	8	0.01128772992	8	1.00000313548
9	111.31677034765	9	0.01269869616	9	9.00000352741

D.	L. Decimals of a Lunation.	M.	L. Decimals of a Lunation.	Th.	L. Decimals of a Lunation.
1	0.0338631897600	1	0.00002351610	1	0.00000000653
2	0.0677263795200	2	0.00004703221	2	0.00000001306
3	0.1015895692800	3	0.00097054831	3	0.00000001959
4	0.1354527590400	4	0.00009406442	4	0.00000002613
5	0.1693159488000	5	0.00011758052	5	0.00000003266
6	0.2031791385600	6	0.00014109662	6	0.00000003919
7	0.2370423283200	7	0.00016461273	7	0.00000004573
8	0.2709055180800	8	0.00018812883	8	0.00000005226
9	0.3047687078400	9	0.00021164494	9	0.00000005879

EXPLANATION.

For tens of years, days, hours, minutes, or ſeconds, remove the decimal point one place forward; for hundreds, two places; for thouſands, three places; and ſo on, as in the annexed example, which is the converſe of the former one.

EXAMPLE.

EXAMPLE.

In 6000 Julian years, 24 days, 4 hours, 59 minutes, 52 ſeconds, *Qu.* How many mean lunations?

		Lun. Dec. of a L.
Years	6000	74211.18023176
Days	20	——0.677263795
	4	——0.135452759
Hours	4	——0.005643865
Minutes	50	——0.001175805
	9	——0.000211645
Seconds	50	——0.000019596
	2	——0.000007784
Anſwer		74212.000000009

This ſhort method may be uſeful in many other caſes; but, as yet, I have only applied it to two. The firſt of which is, to find the number of troy pounds contained in any given number of avoirdupoiſe pounds, and the reverſe. The ſecond is to find the quantity and weight of water that would fill an upright pipe of any given diameter and height: and conſequently, to know what power would be required to work a common pump, or any other hydraulic engine, when the diameter of the bore of the pump, and the height to which the water is to be raiſed, are given; proper allowance being made for friction. Theſe two caſes are as follows.

Troy weight compared with avoirdupoiſe weight.

One troy pound contains 5760 grains; and
One avoirdupoiſe pound contains 7000 grains.
Therefore,
175 troy pounds are equal to 144 avoird. pounds; and
175 troy ounces are equal to 192 avoird. ounces.

On theſe principles, the two following Tables are conſtructed.

1. A Table ſhewing the number of avoirdup. pounds contained in any given number of troy pounds.

Troy Pounds	Avoirdupoiſe pounds and decimals of a pound.
1	0.822857142857143
2	1.645714285714286
3	2.468571428571429
4	3.291428571428572
5	4.114285714285715
6	4.937142857142857
7	5.760000000000000
8	6.582857142857143
9	7.405714285714286

2. A Table ſhewing the number of troy pounds contained in any given number of avoirdupoiſe pounds.

Avoir. Pounds	Troy pounds and decimals of a pound.
1	1.215277777777778
2	2.430555555555556
3	3.645833333333333
4	4.861111111111111
5	6.076388888888889
6	7.291666666666667
7	8.506944444444444
8	9.722222222222222
9	10.937500000000000

EXPLANATION.

For tens of pounds, remove the decimal point one place forward; for hundreds of pounds, two places; for thouſands, three places; and ſo on, as in the following examples.

EXAMPLE

EXAMPLE 1.

In 175 troy pounds, *Qu.* How many avoirdupoiſe pounds?

Troy	Avoirdupoiſe
100	82.2857142857143
70	57.6000000000000
5	4.1142857142857
175	144.0000000000000

Anſwer, 144.

EXAMPLE 2.

In 144 avoirdupoiſe pounds, *Qu.* How many troy pounds?

Av.	Troy
100	121.5277777777778
40	48.6111111111111
4	4.8611111111111
144	175.0000000000000

Anſwer, 175.

When any ſignificant decimal figures remain in the ſum, after integral pounds, they are eaſily reduced into the known parts of a pound: ſeeing that, in troy weight, 24 grains make one penny-weight, 20 penny-weights make an ounce, and 12 ounces make a pound. And, in avoirdupoiſe weight, 16 drams make an ounce, and 16 ounces make a pound.

A Table by which the quantity and weight of water in a cylindrical pipe of any given diameter and perpendicular height may be found.

Diameter of the cylindrical bore one inch.

Feet high.	Quan. of water in cubic inches.	Weight in troy ounces.	In avoirdupoiſe ounces.
1	9.4247781	4.9712340	5.4541539
2	18.8495562	9.9424680	10.9013078
3	28.2743343	14.9137020	16.3624617
4	37.6991124	19.8849360	21.8166156
5	47.1238905	24.8561700	27.2707695
6	56.5486686	29.8274040	32.7249234
7	65.9734467	34.7986380	38.1790773
8	75.3982248	39.7698720	43.6332312
9	84.8230029	44.7411060	49.0873851

EXPLANATION.

For tens of feet high, remove the decimal point one place forward; for hundreds of feet, two places, for thouſands of feet, three places, and ſo on: and you will have the quantity and weight of water in the cylindrical pipe, ſuppoſing its diameter to be one inch.

The contents of cylinders of equal heights are to one another as the ſquares of their diameters. Therefore, having found the quantity and weight of water in a cylinder of one inch bore, you may find the ſame for any other larger bore, by multiplying the above-found quantities by the ſquare of the diameter of the given bore, taken in inches. Thus, ſuppoſe the given bore was 10 inches in diameter (the ſquare of which is 100) and the height of water in the pipe of 10 inches bore was 85 feet, *Qu.* The quantity and weight of the water?

F. high.	Cubic inches.	Troy ounces.	Avoird. ounces.
80	753.982248	397.698720	436.332312
5	47 123890	24.856170	27.270769
85	801.106138	422.554890	493.603081
Mult. by	100	100	100
Anſw.	80110.613800	42255.489000	46360.308100

Which number (80110.6) of cubic inches being divided by 231, gives 346.8 for the number of wine gallons; and the reſpective weights (42255.489 troy ounces, and 46360.308 avoirdupoiſe ounces) being divided, the firſt by 12, and the laſt by 16, give 3521.29 for the number of troy pounds, and 2897.513 for the number of avoirdupoiſe pounds. And ſo much would be required to balance this quantity of water in a pump, or any other hydraulic engine, and as much more to raiſe it as would be ſufficient to overcome the friction of the engine.

The reaſon for removing the decimal points one place forward for tens, two places for hundreds, &c. is evident: for, when any number conſiſts of integer and decimal figures, the ſetting of the decimal point one place forward is the ſame thing as multiplying the number by 10; two places forward, the ſame as multiplying the number by 100; and ſo on: as the tables themſelves are calculated only from

1 to 9. But 1 multiplied by 10 produces 10, and 9 multiplied by 10 produces 90. And ſo on for all the intermediate units.

X. *A Recommendation of* Hadley's *Quadrant for ſurveying, eſpecially the ſurveying of Harbours, together with a particular Application of it in ſome Caſes of Pilotage.*

By the Rev. John Michell, *B. D. F. R. S.*

Read Feb. 14, 1765.

THE uſe of Hadley's quadrant, as an inſtrument to take altitudes at ſea, is already ſo well eſtabliſhed, that it wants no farther recommendation; but there are ſeveral other purpoſes, to which it may be applied, with great advantage, which, though obvious enough, ſeem yet to be hardly ſufficiently attended to. There is no inſtrument ſo well adapted to many kinds of ſurveying, either for exactneſs or conveniency, and particularly the laſt; but the ſurveying of harbours, or ſuch ſands, as lie within ſight of land, may oftentimes be performed by it, not only with vaſtly more eaſe, but alſo with a much greater degree of preciſion, than can be hoped for by any other means, as it is the only inſtrument in uſe, in which neither the exactneſs of the obſervations, nor the eaſe with which they may be taken, are ſenſibly affected by the motion of a veſſel: and hence a ſingle obſerver, in a boat, may generally determine the ſituation of any place he